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Stochastic postponement of the domain transitions and destabilization of current in the Gunn diode

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We use a numerical simulation method to study the nonlinear behavior of spatiotemporal oscillations in the Gunn diode at room temperature, in which the applied bias includes a dc term and a stochastic noise term. We find noise-induced stabilization in the traveling state, stochastic postponement of the stationary- to traveling-domain state, and destabilization of current. Our results show that the temporal noise plays an equivalent role as the spatial characteristic (i.e., Schottky barrier) in the dynamical system. [S1063-651X(98)51202-0]

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Effects of noises in dynamical systems are not only of fundamental interest but also of practical importance, especially in the semiconductor devices. This is because noise, which is usually stochastic in nature, is always present and unavoidable in almost all systems. Effects of noises in nonlinear dynamical systems have been intensively studied for decades [1,2]. One of the striking effects of noise in nonlinear dynamical systems is to induce a shift in the critical value of the control parameter of the system, including physical, chemical, biological, and theoretical model systems [3]. The shift of the critical value of the control parameter, which signals the onset of the instability of the system, may be either toward a smaller critical value (advancement) or toward a larger critical value (postponement). Another striking effect is the controlling of chaos by noise, which had been illustrated in the long Josephson-junction oscillator under the influence of Gaussian random noise [4]. This phenomenon indicates noise-induced stabilization in the dynamical system. In this paper we report a postponement of the critical value of the control paramenter in the simulation study of the transition from a stationary- to a traveling-domain state in n-GaAs (i.e., Gunn diode [5]), in which the electron density at the cathode is considered as a control parameter. We also find that noise tends to stabilize the traveling domain locked in space. But in contrast to the behavior of traveling domain, the current is more irregular in time when the strength of noise is increased.

In a previous work [6], we studied the surface effect in the Gunn diode when the applied dc bias is in the negative differential conductivity (NDC) regime. With the advancement of modern molecular beam epitaxy (MBE) technique, the electron density n at the cathode (metal/n-GaAs contact) may be controlled in experiments, and thus it may be considered as a control parameter. We found that the system may have a transition from a nonoscillating (stationary-domain) state to an oscillating (traveling-domain) state when the control parameter n is increased from $n<n_c$ to $n>n_c$, where n_c is the critical electron density at the cathode. The transition is a global bifurcation known as a nonhysteretic transition caused by saddle-node bifurcation on a limit cycle. In this paper, we study the effect of temporal noise in the saddle-node bifurcation described above. A postponement of the critical value n_c is found in the study. Therefore, temporal noise seemingly plays an equivalent role as the electron density n at the cathode.

We consider an n-type GaAs with a sample length L at room temperature which is applied by a dc bias V. The applied field $E=V/L$ is the NDC region of the current-density field characteristic of n-GaAs. Thus the system may undergo spatiotemporal oscillations. We assume that the only relevant spatial variable is x which is in the direction of the field $\mathbf{E}$. The dynamical equation for $E(x,t)$ can be written as [7]

$$\frac{\partial E}{\partial t}=-\frac{1}{\epsilon}en_0\overline{v}(E)-\overline{v}(E)\frac{\partial E}{\partial x}+\overline{D}\frac{\partial^2 E}{\partial x^2}+\frac{1}{\epsilon}J_{tot}(t), \quad (1)$$

where $\overline{v}$ and $\overline{D}$ are the effective drift velocity and the diffusion constant of the conduction electrons, respectively; J_{tot} is the total current density (conduction current density plus the displacement current density) which is a function of time t only required by the Maxwell equations; and ϵ is the permittivity of the sample.

The dynamical equation (1) is a partial differential equation for the field $E(x,t)$. We use the Fourier series expansion method to treat the spatial dependence of E,

$$E(x,t)=E_s+\sum_m E_m(t)\, e^{imkx}, \quad (2)$$

where $E_s=V/L$ is the applied static uniform field for a noise-free system, E_m are the induced fields where m is an integer extending from $-\infty$ to $+\infty$, and $k=2\pi/L_0$ with L_0 being a length constant. The rate equations for $E_m(t)$ can be obtained by substituting Eq. (2) into Eq. (1), and expanding $\overline{v}(E)$ around the static field E_s to obtain coupled first order ordinary differential equations for E_m,

$$\frac{dE_m}{dt}=(\alpha_m-im\omega)E_m+\frac{1}{\epsilon}(J_{tot}-en_0v_s)\delta_{m,0} - \sum_{m_1+m_2+\cdots+m_p=m}\frac{1}{p!}\beta_{p,m}E_{m_1}E_{m_2}\cdots E_{m_p}, \quad (3)$$

where

$$\alpha_m = -\frac{e}{\epsilon} n_0 v_s^{(1)} - \overline{D} m^2 \left(\frac{2\pi}{L_0}\right)^2,$$

$$\beta_{p,m} = \frac{e}{\epsilon} n_0 v_s^{(p)} + impv_s^{(p-1)} \left(\frac{2\pi}{L_0}\right),$$

$$\omega = \frac{2\pi v_s}{L_0}, \quad v_s = \overline{v}(E_s), \quad v_s^{(p)} = \left.\frac{d^p \overline{v}}{dE^p}\right|_{E=E_s}.$$

By substituting Eq. (2) into the circuit equation, $V = \int_0^L E(x,t)dx$, we obtain a relation between the Fourier components E_m,

$$E_0(t)L + i\sum_{m\neq 0} \frac{L_0}{2m\pi}\left(1 - e^{im(2\pi/L_0)L}\right)E_m = 0. \tag{4}$$

Equations (2)–(4) are the basic equations for our model. Equation (3) is a set of infinitely many coupled first order ordinary differential equations involving dE_m/dt. When the applied field E_s is in the NDC regime, $v_s^{(1)}$ is negative and some of the linear coefficient α_m in Eq. (3) will be positive. This means that some of the mode E_m will be unstable and the system undergoes spatiotemporal oscillations.

As in our previous studies in Gunn diode under dc field [6,8] and rf-alternating field [9,10], we choose nonperiodic basis function $L_0 = 2L$ and the two boundary conditions for the dynamical equation are chosen as $E = V/L$ and $\partial E/\partial x = en_0(n/n_0 - 1)/\epsilon$ at the cathode, where n_0 is the background positive charge density. The electron density n at the cathode is considered as the control parameter. To study the effect of the noise we include a noise term in the applied field E_s in Eq. (2), thus

$$E_s = V/L + A\eta(t), \tag{5}$$

where A is the strength of the noise and $\eta(t)$ satisfies the white noise conditions: $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = \delta(t - t')$. The symbol $\langle\ \rangle$ denotes the ensemble average. The parameter values used for the numerical calculations are $L = 11\ \mu$m, $\overline{v}(E) = [\mu E + v_0(E/E_0)^4]/[1 + (E/E_0)^4]$, $\mu = 5000$ cm^2/Vsec, $v_0 = 8.5\times 10^6$ cm/sec, $E_0 = 4.0$ kV/cm, $\epsilon = 1.1\times 10^{-12}$ Coul/Vcm, $n_0 = 10^{15}$/cm^3, $\overline{D} = 140$ cm^2/sec, and $V/L = 4.8$ kV/cm, which is in the NDC regime. With this field it is sufficient to keep the modes E_m with $|m| \leq 3$ and all higher order modes can be neglected as $\alpha_m < 0$ for $|m| \geq 3$.

When there is no noise, the system undergoes a transition from a stationary-domain state to a traveling-domain state when the control parameter n exceeds the critical value $n_c = 0.73n_0$. In the following we present the numerical results of the noise effect by substituting Eq. (5) into Eq. (2) and solve Eqs. (3) and (4) numerically by keeping only E_m with $|m| \leq 3$. We first study the noise effect in the traveling-domain state which is shown in Fig. 1, where the **X** denotes the position of the maximum field in the high-field domain and the successive horizontal curves in Figs. 1(a) and 1(b) mean that when **X** reaches the right end of the sample, it will return to the left end as time goes on. For a noise-free system [Fig. 1(a)], the high-field domain travels with almost a constant velocity from the cathode to the anode except that the velocity of the high-field domain will slow down in some region of the sample. The time duration of slowing down depends on the electron density n at the cathode [6]. When the stochastic noise with small A is added to the system, the traveling domain has the tendency to be locked in a certain

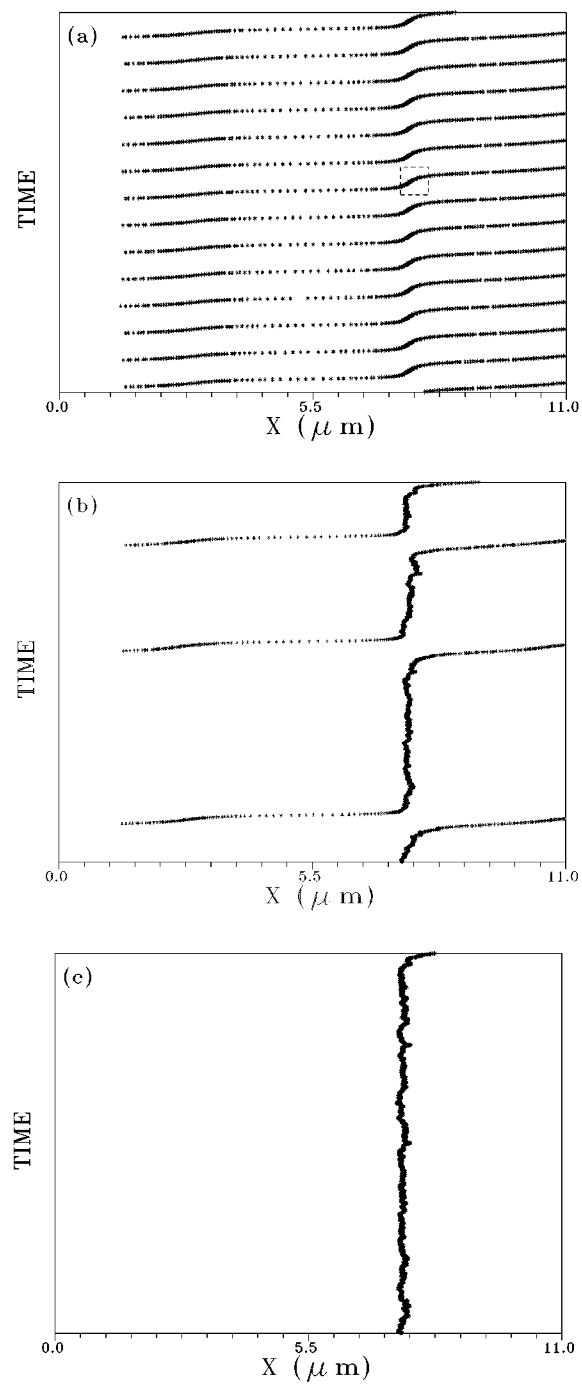

FIG. 1. Nonlinear response of the traveling domains in Gunn diode under different values of noise strength A: (a) $A = 0$ V/cm, (b) $A = 380$ V/cm, and (c) $A = 400$ V/cm. In this example, the nonideal Schottky barrier at the cathode (i.e., $n/n_0 = 0.9$) is used, and the total time interval is equal to 4.125 ns. For clear illustration, we plot time versus the position of the maximum field **X** in the high-field domain. The dashed square in (a) indicates the velocity of electric-field domain slowdown in sample.

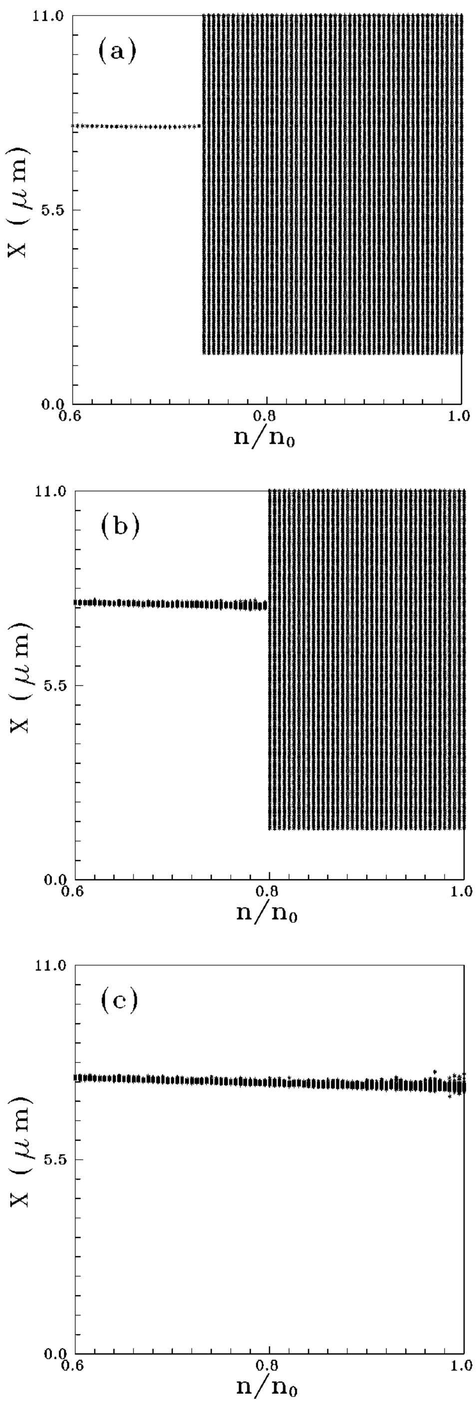

FIG. 2. Position of the maximum field **X** in the high-field domain versus n/n_0 for different values of A: (a) $A=0$ V/cm, (b) $A=240$ V/cm, and (c) $A=400$ V/cm.

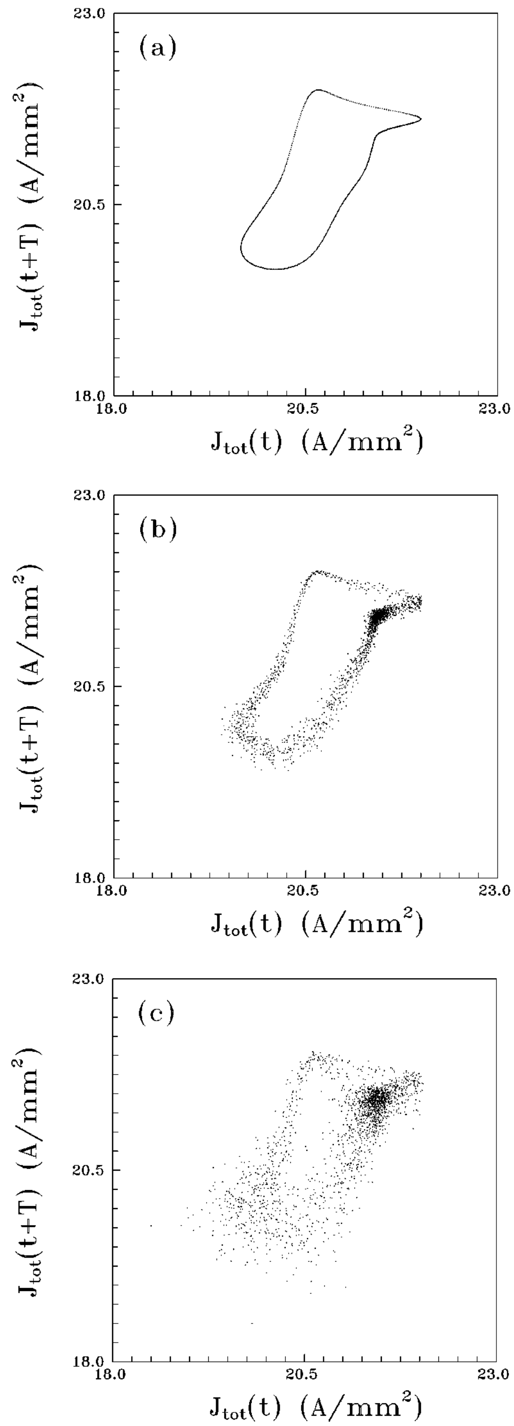

FIG. 3. Attractor reconstructions obtained by the time-delay method (T=20.6 ps) from current oscillations at different values of A: (a) $A=0$ V/cm, (b) $A=40$ V/cm, and (c) $A=120$ V/cm. The control parameter n/n_0 is 0.9.

narrow region of the sample for a short duration of time [Fig. 1(b)], during which the average traveling velocity is almost zero. The average time duration of being locked increases as the noise strength A increases. Finally the traveling domains become entirely locked in that narrow region of the sample when the noise strength A is sufficiently high [Fig. 1(c)]. This result implies that the transit characteristic in a noise-free Gunn diode can be entirely destroyed to result in a stationary characteristic due to an external noise.

Another important point of our study is that noise postpones the transition from the stationary-domain state to the traveling-domain state as the control parameter n increases. For a noise-free system the critical value for the transition is $n_c=0.73n_0$, but n_c increases as the noise strength increases. In Fig. 2 we plot the position of the maximum field **X** in the high-field domain as a function of n/n_0 with several noise strength A. In Fig. 2(a) we show that the critical value for the transition is $n_c=0.73n_0$ for a noise-free system. When the noise strength A is 240 V/cm, we can see that $n_c = 0.8n_0$ and the position fluctuation in the stationary-domain state [Fig. 2(b)]. In Fig. 2(c) when the noise strength A is increased to 400 V/cm, the traveling-domain state disappears and become entirely locked in narrow region of the sample. The illustration in Fig. 2 is called stochastic postponement.

Finally, we want to analyze noise-induced destabilization via attractor reconstructions by the time delay method [11] in Fig. 3. For a noise-free system, periodic current oscillation in the traveling-domain state is shown in Fig. 3(a). The oscillating frequency is equal to 8.6 GHz. When the noise (i.e., $A = 40$ V/cm) is added to the traveling-domain state, a small fluctuation of periodic current oscillation appears [Fig. 3(b)]. When the noise is increased to 120 V/cm, the current oscillation becomes erratic in a wider regime [Fig. 3(c)]. Therefore, in contrast to the above-mentioned noise-induced stabilization from the traveling domain to the stationary (or locked) domain, the current oscillation will become more stochastic and the amplitude of oscillation will become larger than the noise-free case when noise is added to the system. This phenomenon is called noise-induced destabilization from periodic current oscillation to erratic current oscillation.

The temporal noise plays an equivalent role as the spatial characteristic (i.e., Schottky barrier) in the dynamical system which may be analyzed in terms of the theory of nonlinear dynamics. With only a dc bias in the NDC regime, the dynamical behavior of the Gunn diode can be described as the saddle-node bifurcation on a limit cycle, in which the control parameter is the electron density n at the cathode. The system consists of unstable focus, stable node, saddle node, and a large-amplitude limit cycle. The unstable focus, the stable node, and the large-amplitude limit cycle correspond, respectively, to the unstable spatially uniform state, the stationary-domain state, and the traveling-domain state. The state of the Gunn diode (stationary- or traveling-domain) depends on whether or not the stable node and the saddle node collide. When the stable node and the saddle node meet together (i.e., $n > n_c$), there is no other fixed point on the large-amplitude limit cycle, then the traveling-domain state appears. When there is white noise in the traveling-domain state, the pair fixed points, i.e., the stable node and the saddle node, are created on the large-amplitude limit cycle, then the traveling domain will be locked in space. This situation is quite different from the well-known stochastic Hopf bifurcation, in which noise makes the unstable focus stabilized. But in contrast to the noise-induced stabilization in the traveling domain, we find the noise-induced destabilization in the oscillating current. The reason can be understood via the above explanations. When noise strength is large enough to lock the traveling domain in the space, it means the whole E_m modes will tend to some fixed values and the total current density J_{tot} is entirely determined by the noise terms in Eq. (3). Thus, the periodic current oscillation will be replaced by the erratic current oscillation when noise is added to the dynamical system.

In conclusion, we numerically study the spatiotemporal structures in the Gunn diode under externally white noise. The stochastic postponement of the transition from a stationary- to a traveling-domain state, noise-induced stabilization for electric-field domain and noise-induced destabilization for oscillating current are found. This paper reports that noise-induced stabilization may be found in a semiconductor device, e.g., a Gunn diode. Our result is quite unique and rather different from earlier studies about the effects of noise. We hope that these numerical results can be experimentally observed.

ACKNOWLEDGMENTS

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